

Tutorial 3

In this tutorial we will develop some intuition about biorthogonal bases.

Construction of “random” biorthogonal basis:

- 1.) Construct a biorthogonal basis $\{u_1, u_2\}$ for \mathbb{R}^2 by generating two random vectors in the space.
- 2.) Construct the dual basis $\{\tilde{u}_1, \tilde{u}_2\}$ for $\{u_1, u_2\}$.
- 3.) Show numerically that one can use the biorthogonal basis pair just constructed to reconstruct an arbitrary vector.
- 4.) What happens when one interchanges primary and dual basis?
- 5.) Plot the basis $\{u_1, u_2\}$ and its dual $\{\tilde{u}_1, \tilde{u}_2\}$ using the provided routine `plotVectors2D()`.

Structure of dual bases:

- 1.) Construct a biorthogonal basis pair $(\{u_1, u_2\}, \{\tilde{u}_1, \tilde{u}_2\})$ where u_1 and u_2 have an angle of $\alpha = 90^\circ, \dots, 180^\circ$ and are of unit length.
- 2.) Plot the biorthogonal basis pair for each angle.
- 3.) Does the error for reconstructing an arbitrary vector change as a function of the angle? Explain your observation using the properties of the basis matrix.