Mathematical Tools for Computer Graphics 2017

Tutorial 6

In this tutorial we will study how to work numerically with frames.

1.) The Mercedes-Benz frame is given by

$$u_1 = \big(0.0, 1.0\big)^T \tag{1a}$$

$$u_2 = \big(\sqrt{3}/2, -1/2\big)^T \tag{1b}$$

$$u_3 = \big(-\sqrt{3}/2, -1/2\big)^T \tag{1c}$$

Use the function `plotVectors2D()` in the provided source code to plot the frame.

2.) Construct the (canonical) dual frame to the Mercedes Benz frame given in Eq. 1. Plot the dual frame vectors.

3.) The Mercedes Benz frame in Eq. 1 is an example of a tight frame. These can be considered as generalizations of orthogonal bases. Explain the last statement.

4.) The overcomplete generalization of an orthonormal basis is a Parseval (tight) frame. Modify the Mercedes Benz frame in Eq. 1 so that it becomes Parseval tight.

5.) In the following we will study how the redundancy of a frame affects the reconstruction error that can be attained. Consider $\mathbb{R}^{100}$ and generate random frames with a redundancy ranging from 1 to 3. For each redundancy determine the average reconstruction error for a set of random vectors in the space. Plot the average errors as a function of the redundancy. How could the reconstruction error for fixed redundancy be improved?