A Markov Model for Multi-Criteria Multi-Person Decision Making

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Abstract

This paper describes a new algorithm for the evaluation of alternatives by a group of decision makers according to multiple criteria. The algorithm is motivated by the need to quickly evaluate a large number of ideas in the early stages of an innovation process, when little or no information about the ideas is available. The algorithm is based on a Markov chain model which is derived from pairwise comparisons of ideas. The steady-state solution of this Markov chain yields a ranking vector for the alternatives. The algorithm is similar to the "PageRank" method used by Google. The new algorithm does not require absolute values and allows assignment of weights both to the decision makers and to the evaluation criteria.

1. Introduction

We consider the problem of a group of decision makers who have to select one or more alternatives which best fulfil a given set of decision criteria. We are especially interested in the case which is characterised by the following attributes:

- Little or no information is available about the alternatives; the decision makers base their evaluations on intuition or guesswork.
- The decision makers may give inconsistent evaluations, both with respect to their own evaluations and with respect to each other.
- It may be desirable to assign different degrees of importance to the decision makers and the decision criteria.
- It may be acceptable to trade accuracy for complexity, i.e. to accept a larger number of winning alternatives than were specified, if this can be achieved at reduced cost.
- A large number of alternatives and/or evaluation criteria may be involved.

This situation is typical in the early stages of an innovation process. Modern innovation processes are typically modelled on a stage-gate format [1]. At each gate, the projects are selected which will be pursued in the next stage. Typically, little information is available at the outset, and evaluations are based solely on intuition and experience.

Often, a large number of ideas are input into the process, and the number of decision criteria may also be relatively large. The goal of the early stages of the innovation process is to select the best ideas to keep in the innovation process; at the first gate, this may be up to 10% of the total number of ideas.

The design goals for our algorithm were

- that it make use of pairwise comparisons (rather than ordinal values),
- that it have a simple mathematical structure, in order to be attractive to practitioners,
- that it allow a simple computer interface requiring no algorithmic expertise in order to be used effectively,
- that it be able to account for differences in experience or knowledge between the decision makers.
- that it allow evaluation results of different decision makers and criteria to be selected and aggregated,
- that it be amenable to sensitivity analysis.

2. Background

2.1 Multi-criteria decision making

Multi-criteria decision making (MCDM) is a discipline aimed at supporting decision makers who are faced with the evaluation of many alternatives with respect to several criteria [2], [3], [4]. Depending on which type of result is needed, many different MCDM methods are available. Thirty available methods are discussed, for example, in [5].

In multi-person decision making (MPDM), more than one person is involved in the decision making process. Because most MCDM methods assume only one decision maker, strategies for mapping several opinions onto a single result are needed [6], [7], [8].

In the approach described in this paper, the decision makers perform independent partial evaluations which are subsequently combined to obtain an overall set of evaluations, which form the basis of the ranking computation.

2.2. The AHP and WISDOM methods

Given the applications we are interested in, the two most appropriate MCDM methods for comparison purposes are AHP and WISDOM [9].

The Analytic Hierarchy Process (AHP) [10] is based on a hierarchy of evaluation criteria, and uses paired comparisons of alternatives with respect to these criteria. Gradations in the comparisons are expressed using numerical values. The ranking of the alternatives is obtained from an eigenvalue computation on a suitably aggregated matrix.

Known drawbacks of the AHP method are the need to deal with inconsistent sets of evaluations, the large number of pairwise comparisons needed, a complex mathematical model which is intransparent to the user and questionable rankings resulting from innocuous individual comparisons. The incomplete AHP method tries to use a smaller number of pairwise comparisons without compromising the result [11]

The Weightless Incremental Selection and Ordering Method (WISDOM) [12] is also based on paired comparisons with gradations. However, in this case, the gradations are qualitative, rather than quantitative. The evaluation criteria are also weighted using pairwise comparisons. An algebra for computations on these qualitative relations is then developed for which an iterative method is used to compute a ranking.

WISDOM shares many of the positive attributes of AHP. However, it also has several advantages over AHP. The first is the use of qualitative expressions rather than numerical ones for the pairwise comparisons and the computations. It is also able to explicitly detect evaluation inconsistencies. Its main disadvantage – as with AHP – is the lack of a natural extension to multi-user applications. One such extension, named TeamWISDOM, was presented by Weber [9].

Our method is similar to both AHP and WISDOM, in that it is also based on pairwise comparisons of the alternatives with respect to different criteria and that it subsequently computes a ranking vector. Like AHP, our method also performs an eigenvalue computation on a matrix representing the results of the individual comparisons. However, in contrast to both methods, ours does not provide for differing weights in the pairwise comparisons (although this extension would be easy to accommodate), and its overall structure is simpler.

2.3. Markov chains

Discrete-time Markov chains (DTMCs) are well-researched mathematical models with many applications in Science and Engineering. A DTMC is described by a stochastic matrix P and a probability vector π. The steady-state solution of the DTMC contains the probabilities of each of the system states and is given by the solution of the linear system of equations

$$\pi P = \pi \tag{1}$$

Markov chains are drawn as directed, annotated graphs, where the nodes represent the states and the arcs the possible state transitions. The weights associated with the arcs describe the one-step probabilities for each state transition. A state or set of states of a Markov chain is called absorbing, if it contains only incoming arcs.

3. The model

Our basic premise is that that evaluation judgements made by the participants are not absolutes based on quantitative information, but are subject to uncertainty. For this reason, we represent them in our model by probabilities and interpret the resulting ranking as a vector of probabilities.

Our method is similar to the PageRank algorithm [13], which is used to estimate the relative importance of hyperlinked documents. The most well-known application of the PageRank algorithm is Google's use of it to compute the ranking of search results in the Internet. This ranking is also a vector of probabilities.

3.1. Basic evaluation step

The atomic evaluation step in the method is for participant p_k to state that Alternative a_{m1} is superior to Alternative a_{m2} with respect to Criterion d_l. This we will denote as follows:

$$p_k(d_l) : a_{m1} > a_{m2} \tag{2}$$

Each evaluation is assigned a weight between 0 und 1 and mapped to a coefficient of a stochastic matrix as described in Section 3.2. This results in a discrete-time Markov chain, whose steady-state solution vector represents a ranking of the alternatives.

3.2. Mathematical description

We denote the participants in the decision process by p_k with $k = 1\ldots K$ and the decision criteria by d_l, with $l = 1\ldots L$. The alternatives to be evaluated are denoted by a_m, with $m = 1...M$.

We define a matrix A of dimension $K\times L$ whose coefficients α_{kl} satisfy $0 \leq \alpha_{kl} \leq 1$ and

$$\sum_{k=1}^{k=K}\sum_{l=1}^{l=L}\alpha_{kl} = 1 \qquad (3)$$

These coefficients are used to assign weights to the evaluations made by the participants. Thus the coefficient α_{kl} contains information about the level of expertise of participant p_k with respect to Criterion d_l as well as the degree of importance of Criterion d_l, where larger values imply greater importance.

We next define stochastic matrices P_{kl} to represent the evaluations made by participant p_k with respect to decision Criterion d_l. Each matrix P_{kl} is constructed by first setting

$$\left(P_{kl}\right)_{m1,m2} = \begin{cases} \dfrac{1}{\delta_{m1}+1} & if \;\; p_k(d_l): a_{m2} > a_{m1} \\ 0 & otherwise \end{cases} \qquad (4)$$

where δ_{m1} is the number of non-zero entries in the $m1$-th row of matrix P_{kl}. Finally, the main diagonal coefficients are set to satisfy

$$\left(P_{kl}\right)_{m1,m1} = 1 - \sum_{\substack{m2=1 \\ m2\neq m1}}^{m2=L} (P_{kl})_{m1,m2} \qquad (5)$$

Each matrix P_{kl} is stochastic. The following weighted sum of the P_{kl} yields a stochastic matrix P_{eval} which represents the complete set of evaluations by all decision makers with respect to all criteria:

$$P_{eval} = \sum_{k=1}^{k=K}\sum_{l=1}^{l=L}\alpha_{kl}\, P_{kl} \qquad (6)$$

The matrix P_{eval} cannot be used to compute the ranking vector directly, since it may contain absorbing states, which would absorb all the probability. To solve this problem, we define a stochastic matrix P_{fill}, each coefficient of which has the value $1/M$ and define the matrix P as

$$P = (1-\varepsilon)\, P_{eval} + \varepsilon\, P_{fill} \qquad (7)$$

using a parameter ε with $0 < \varepsilon << 1$. The matrix P is stochastic and irreducible, and therefore defines a Markov chain with no absorbing states.

3.3 The evaluation algorithm

The evaluation algorithm for computing a ranking vector is given by a sequence of five steps:

1. Choose the values α_{kl}.
2. Choose a value for ε.
3. Enter all participants' comparisons as coefficients into the appropriate matrices P_{kl}.
4. Compute P according to Equation (7).
5. Solve Equation (1) to obtain π.

After termination of the algorithm, the rank of Alternative a_m is determined by the corresponding value π_m in the probability vector π, whereby larger values represent higher ranks.

The number T of pairwise evaluations is given by

$$T = \frac{K \cdot L \cdot M \cdot (M-1)}{2} \qquad (8)$$

when every participant makes every possible pairwise comparison with respect to every criterion. T may be prohibitively large when M is large. In practice, we hope that a reliable ranking vector can be achieved with a smaller number of comparisons.

We currently have no heuristic for determining a suitable value for ε. Since the only role of ε is to prevent the matrix P from becoming reducible, we assume that it is sufficient to set ε to an arbitrary very small number. Experiments suggest that the influence of ε on the computed ranking is negligible.

The PageRank model interprets the surfing behaviour of a user in the Internet as a random walk, converting the hyperlinks between documents into probabilities. This corresponds to the pairwise comparisons in our case. The possibility of directly entering a URL into the browser corresponds to the matrix P_{fill} in our model.

4. Example model

As a simple example we consider a small decision problem consisting of three alternatives, three criteria and three decision makers.

The weighting matrix A_1 is given by

$$A_1 = \begin{bmatrix} 0.125 & 0.1 & 0.1 \\ 0.125 & 0.1 & 0.075 \\ 0.25 & 0.05 & 0.075 \end{bmatrix} \qquad (9)$$

We assume the full set of 27 comparisons is made, yielding the following nine matrices P_{kl}:

$$P_{11} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \end{bmatrix}; \quad P_{12} = \begin{bmatrix} 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}; \quad P_{13} = \begin{bmatrix} 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \\ 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix};$$

$$P_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \end{bmatrix}; \quad P_{22} = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0 \\ 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \end{bmatrix}; \quad P_{23} = \begin{bmatrix} 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \\ 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix};$$

$$P_{31} = \begin{bmatrix} 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \\ 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}; \quad P_{32} = \begin{bmatrix} 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \\ 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix}; \quad P_{33} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0.\bar{3} & 0.\bar{3} & 0.\bar{3} \\ 0 & 0 & 1 \end{bmatrix};$$

(10)

After choosing ε = 0.1, we obtain the following evaluation matrix P from Equation (7):

$$P = \begin{bmatrix} 0.50 & 0.26 & 0.24 \\ 0.16 & 0.74 & 0.10 \\ 0.14 & 0.35 & 0.51 \end{bmatrix} \qquad (11)$$

which is also shown graphically in Figure 1.

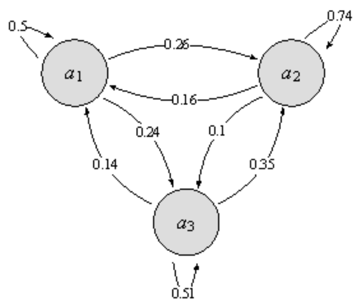

Figure 1: Markov chain for example model

5. Example experiment

In this section we demonstrate that our algorithm creates a ranking of the alternatives for different criteria and different decision makers.

The solution vector for the example model is

$$\pi_1 = (\pi_{a1}, \pi_{a2}, \pi_{a3}) = (0.23,\ 0.54,\ 0.22) \qquad (12)$$

We are interested in the development of the ranking vector as the evaluations are submitted. To obtain this result, we compute the matrix P and solve the resulting Markov chain after each comparison has been made. The development of the probabilities is shown in Figure 2. The initial probabilities are set to:

$$\pi_1 = (0.3\bar{3},\ 0.3\bar{3},\ 0.3\bar{3}) \qquad (13)$$

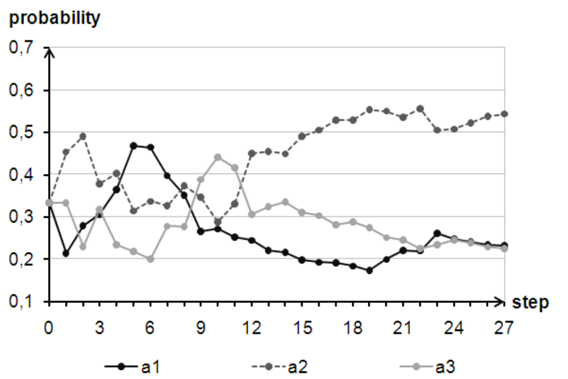

Figure 2: Probability values after each comparison

In the subsequent steps, the probabilities change rapidly. After comparison 12, a clear tendency becomes apparent, which enables the final ranking to be predicted safely. However, this is not necessarily true in all cases, and a method is needed to determine an ordering for the evaluations which produces the final ranking as quickly as possible.

This experiment shows that the algorithm described enables the evaluation of alternatives by different decision makers with respect to different criteria. The vector obtained represents a ranking of the alternatives. Assigned weights allow the modelling of different degrees of importance both of the criteria and the decision makers.

6. Summary and outlook

This paper describes a new algorithm for multi-person multi-criteria decision making based on a Markov chain model. With one exception, the algorithm fulfils all the specified design goals derived from application in evaluating ideas in the early stages of an innovation process. The exception concerns the convergence of the ranking vector as individual evaluations are input. The algorithm thus promises to be useful, at least in this application area.

The advantages we believe our approach possesses include simplicity of usage and in the mathematical model, the ability to represent levels of competence of the decision makers with respect to the various criteria, the tolerance of inconsistencies and the ability to "mix and match" different combinations of evaluations.

An alternative to pre-assigning weights to the evaluation criteria would be to subject these to the same pairwise comparisons as the alternatives. This is the approach used both by AHP and WISDOM. In this case, the relative importance of the criteria would be a group decision, instead of a predefined quantity. This approach could be applied analogously to the participants' coefficients.

In future research we want to minimise the number of evaluations needed to obtain an accurate ranking result. This means finding heuristics for ordering the comparisons. One approach may be to compute the sensitivity of the ranking vector with respect to each possible judgement. Then, the algorithm can prompt the decision makers to input the judgement that is expected to have the strongest positive effect on the ranking vector.

Further work will also include developing a method for enforcing irreducibility which retains sparsity, the implementation of comparisons of the form "much better than", comparing the results with those obtained from other methods, determining the intersubjectivity of the computed ranking among the decision makers and studying the behaviour of the method using data from real-life problems.

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